AP Calculus BC

Unit 9 - Sequences and Series

AP Calculus BC – Worksheet 72 **Intro to Infinite Sequences**

1	Find the first four terms and the 50th term for the sequence: $a_n = \dfrac{n}{n+7}$
2	Find the first six terms and the 50th term for the sequence: $d_n = n^2 - 2n$.
3	Find the first four terms and the 8th term for the sequence: $a_1 = 1;\ a_n = 2a_{n-1}$ for all $n \geq 2$.
4	Find the first four terms and the 8th term for the sequence: $u_1 = 1; u_2 = 2;\ u_n = u_{n-1} + u_{n-2}$ for all $n \geq 3$.
5	Determine the convergence or divergence of $a_n = \dfrac{2n+1}{n}$. If the sequence converges, find its limit.
6	Determine the convergence or divergence of $a_n = (-1)^n \dfrac{n-1}{n+1}$. If the sequence converges, find its limit.
7	Determine the convergence or divergence of $a_n = 5 + (0.9)^n$. If the sequence converges, find its limit.
8	Determine the convergence or divergence of $a_n = n \sin\left(\dfrac{7}{n}\right)$. If the sequence converges, find its limit.
9	Find the limit of the sequence $\left\{\dfrac{3n^4}{n^4+1}\right\}$ or state that it does not exist.
10	Find the limit of the sequence $\left\{\dfrac{5e^n+1}{e^n}\right\}$ or state that it does not exist.

AP Calculus BC – Worksheet 73 **Introduction to Sequences – Day 2**

Evaluate each limit or state that it does not exist.

1) $\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^{11x}$	2) $\lim_{x\to1}\frac{\int_1^x \cos(5t)\,dt}{x^2-1}$	3) $\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^{n}$

Find the limit of each sequence or determine that the limit does not exist.

4) $a_n=\sqrt{\left(1-\frac{1}{10n}\right)^{n}}$	5) $a_n=\left(1+\frac{9}{n}\right)^{8n}$	6) $a_n=\frac{n}{e^n+11n}$
7) $a_n=\left(\frac{10}{n}\right)^{10/n}$	8) $a_n=\ln\left(n^3+5\right)-\ln\left(7n^3+11n\right)$	9) $a_n=\left(n\sin\frac{17}{n}\right)$
10) $a_n=\frac{(-1)^n n}{2n+3}$	11) $a_n=\cos\left(\frac{n\pi}{12}\right)$	12) $a_n=\frac{(-1)^{n+1}}{4n+1}$
13) $a_n=\frac{3^n}{3^n+5^n}$	14) $a_n=2+\cos\frac{5}{n}$	15) $a_n=\frac{\ln n}{n^{1.6}}$
16) $a_n=\frac{8^n+5^n}{8^n+n^{110}}$	17) $a_n=\frac{9^n}{n^9 7^n}$	

Answers

1) e^{-11}	2) $\frac{\cos 5}{2}$	3) e^2	4) $e^{-1/20}$	5) e^{72}	6) 0
7) 1	8) $-\ln 7$	9) 17	10) DNE	11) DNE	12) 0
13) 0	14) 3	15) 0	16) 1	17) DNE	

AP Calculus BC – Worksheet 74 **Sequences and Series**

1	Write an expression for the nth term. (a) 3, 8, 13, 18, . . . (b) 5, -15, 45, -135, . . . (c) 1, 4, 9, 16, 25, . . .
2	Use the nth Term Divergence Test to determine whether or not the following series converge: (a) $\sum_{n=1}^{\infty}\frac{1+3n^2+n^3}{4n^3-5n+2}$ (b) $\sum_{n=1}^{\infty}\frac{1}{n^2}$ (c) $\sum_{n=1}^{\infty}\frac{n!}{2n!+1}$ (d) $\sum_{n=1}^{\infty}\frac{(n+2)!}{10n!}$
3	Use the indicated test for convergence to determine if the series converges or diverges. If possible, state the value to which it converges. (a) Geometric Series: $3+\frac{15}{4}+\frac{75}{16}+\frac{375}{64}+\cdots$ (b) Geometric Series: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\cdots$
4	Determine whether the following series converge or diverge. If they converge, find their sum. (a) $\sum_{n=1}^{\infty}\frac{2n+3}{3n-5}$ (b) $\sum_{n=1}^{\infty}\frac{n!}{2n!+1}$ (c) $\sum_{n=1}^{\infty}\frac{3^n-2}{3^n}$ (d) $\sum_{n=2}^{\infty}\frac{1}{(1.1)^n}$

AP Calculus BC – Worksheet 75 **p-series and Integral Test**

Determine if each series converges or diverges. If it converges, find the sum.

1) $\frac{1}{17}+\frac{1}{289}+\frac{1}{4913}+\ldots+\frac{1}{17^k}+\ldots$	2) $1+\frac{1}{2}+\left(\frac{1}{2}\right)^2+\ldots+\left(\frac{1}{2}\right)^n+\ldots$	3) $\sum_{n=0}^{\infty}\frac{5^n}{9^{n+1}}$

Use the divergence test (n^{th} term test) to determine whether each series diverges or state that the test is inconclusive.

4) $\sum_{n=0}^{\infty}\frac{n}{5n+1}$	5) $\sum_{k=0}^{\infty}\frac{3}{900+3k}$

Determine the convergence or divergence of each series.

6) $\sum_{k=2}^{\infty}\frac{1}{(k-1)^3}$	7) $\sum_{k=3}^{\infty}\frac{1}{(k-2)^4}$	8) $\sum_{n=1}^{\infty}\frac{12}{n^{1/2}}$
9) $\sum_{k=2}^{\infty}\frac{1}{6e^k}$	10) $\sum_{k=0}^{\infty}\frac{6}{\sqrt{k+3}}$	

Answers

1) Converges; $\frac{1}{16}$	2) Converges; 2	3) Converges; $\frac{1}{4}$	4) Diverges by nth term test	5) Inconclusive
6) Converges by p-series test	7) Converges by p-series test	8) Diverges by p-series test	9) Converges by Integral test	10) Diverges by integral test

AP Calculus BC – Worksheet 76 **Convergence of Infinite Series**

Write out the first four terms of the sequence of partial sums for each geometric series. Then find the sum of the infinite series.

1. $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n}$	2. $\sum_{n=1}^{\infty} \frac{7}{4^n}$	3. $\sum_{n=0}^{\infty} (-1)^n \frac{5}{4^n}$	4. $\sum_{n=0}^{\infty} \left(\frac{2^{n+1}}{5^n}\right)$

Determine whether each series converges or diverges. Give a reason for your answer. If the series converges, find its sum, if possible.

5. $\sum_{n=0}^{\infty} \left(\frac{1}{\sqrt{2}}\right)^n$	6. $\sum_{n=0}^{\infty} \cos(n\pi)$	7. $\sum_{n=1}^{\infty} \ln\frac{1}{n}$	8. $\sum_{n=0}^{\infty} \frac{n!}{1000^n}$
9. $\sum_{n=0}^{\infty} \left(\frac{e}{\pi}\right)^n$	10. $\sum_{n=1}^{\infty} \left(\frac{1}{10}\right)^n$	11. $\sum_{n=1}^{\infty} \frac{n}{n+1}$	12. $\sum_{n=2}^{\infty} \frac{5}{n+1}$
13. $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n}}$	14. $\sum_{n=1}^{\infty} \frac{-2}{n\sqrt{n}}$	15. $\sum_{n=1}^{\infty} \left(-\frac{1}{8^n}\right)$	16. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$
17. $\sum_{n=1}^{\infty} \frac{2^n}{3^n}$	18. $\sum_{n=0}^{\infty} \frac{-2}{n+1}$	19. $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$	20. $\sum_{n=1}^{\infty} \frac{e^n}{1+e^{2n}}$

AP Calculus BC – Worksheet 77 **Sequences and Series Review**

1) $\sum_{k=0}^{\infty}\frac{2}{6^{k+1}}=$	2) $\sum_{k=0}^{\infty}\frac{3}{10^{k+1}}=$
3) List the first six terms for the recursively defined sequence: $a_1=1,\ a_2=3,\ a_n=2a_{n-1}+3$	4) List the first six terms for the recursively defined sequence: $a_1=1,\ a_2=3,\ a_n=a_{n-1}+a_{n-2}$
5) Find the formula for the n^{th} term of the sequence given by: $1,\frac{1}{3},\frac{1}{5},\frac{1}{7},\frac{1}{9}\ldots$	6) Find the formula for the n^{th} term of the sequence is given by: $1,-\frac{1}{3},\frac{1}{5},-\frac{1}{7},\frac{1}{9}\ldots$
7) Determine if $\sum_{n=1}^{\infty}\frac{1}{\left(\sqrt{5}-1\right)^n}$ converges or diverges.	8) Determine if $\sum_{n=1}^{\infty}\frac{1}{\left(\sqrt{3}-1\right)^n}$ converges or diverges.
9) The integral test confirms that the series $\sum_{n=1}^{\infty}\frac{e^{1/n}}{n^2}$ converges. What is $\int_1^{\infty}\frac{e^{1/x}}{x^2}\,dx$?	
10) Determine the convergence or divergence of each series: (a) $\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n$ (b) $\sum_{n=1}^{\infty}\frac{1}{n}$ (c) $\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ (d) $\sum_{n=1}^{\infty}\frac{1}{n^2}$ (e) $\sum_{n=1}^{\infty}\frac{1}{e^n}$ (f) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ (g) $\sum_{n=1}^{\infty}n$ (h) $\sum_{n=1}^{\infty}\left(\frac{3}{2}\right)^n$	
11) Find the sum of the series: $\sum_{n=1}^{\infty}\frac{1}{2^{n-1}}$	12) Find the sum of the series: $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{3^{n-1}}$